





UNIVERSITY OF  
ILLINOIS LIBRARY  
AT URBANA-CHAMPAIGN  
BOOKSTACKS

64 E 2-

## CENTRAL CIRCULATION BOOKSTACKS

The person charging this material is responsible for its renewal or its return to the library from which it was borrowed on or before the **Latest Date** stamped below. **You may be charged a minimum fee of \$75.00 for each lost book.**

Theft, mutilation, and underlining of books are reasons for disciplinary action and may result in dismissal from the University.

TO RENEW CALL TELEPHONE CENTER, 333-8400


UNIVERSITY OF ILLINOIS LIBRARY AT URBANA-CHAMPAIGN

APR 29 1998

AUG 03 1998

When renewing by phone, write new due date below previous due date.

L162



Digitized by the Internet Archive  
in 2011 with funding from  
University of Illinois Urbana-Champaign

<http://www.archive.org/details/hotellingrentsin1629kols>

Hotelling Rents in Hotelling Space: Exhaustible  
Resource Rents with Product Differentiation

*Charles D. Kolstad*

The Library of the

MAR 24 1990

University of Illinois  
of Urbana-Champaign

WORKING PAPER SERIES ON THE POLITICAL ECONOMY OF INSTITUTIONS NO. 34



College of Commerce and Business Administration  
Bureau of Economic and Business Research  
University of Illinois Urbana-Champaign



# BEBR

FACULTY WORKING PAPER NO. 90-1629

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

February 1990

Hotelling Rents in Hotelling Space:  
Exhaustible Resource Rents with Product Differentiation

Charles D. Kolstad\*  
Department of Economics

\*University of Illinois, 1206 South Sixth Street, Champaign, IL 61820.  
Comments from John Braden and from workshop participants at the University of Illinois have been appreciated.





### Abstract

Using a Hotelling spatial model, this paper examines the inter-relationship between resource rents in related exhaustible resource markets. In a product space context, we show that even if two exhaustible resources are not currently in the same market (e.g., coal and oil), differential scarcity will link their current scarcity rents. This applies analogously in geographic markets. Another result is that monopoly accelerates depletion of substitutes and may not affect time to depletion of the monopolized resource.



## I. INTRODUCTION

Three fundamental results in the economics of exhaustible resources are: (a) resource rents rise at the rate of interest and depend on resource demand and the initial stock of the resource; (b) monopoly in an exhaustible resource increases price, depresses initial production rates and extends time to exhaustion; and (c) higher grade resources are extracted first. In this paper we show that all three of these conclusions must be modified in markets involving multiple exhaustible resources that are not perfect substitutes for each other.

We show that rent for an exhaustible resource depends on costs, demand, and stocks of that resource as well as costs, demand, and stocks of substitute resources. Ignoring stocks of substitutes will lead to an over-statement of rents. Thus the Hotelling rent associated with oil depends on reserves of oil, gas, and coal. In a spatial context, the rent on coal in Kentucky depends on reserves in Kentucky and elsewhere, even reserves in Australia. Although the essence of economics is that "everything is interrelated," this direct connection between rents for resources that are not perfectly substitutable is usually overlooked.<sup>1</sup>

Furthermore, the maxim that "monopoly is the conservationist's best friend" will not necessarily apply to goods that are substitutes for the monopolized good. Certainly OPEC has effectively conserved the

---

<sup>1</sup>For instance, the 1970s debate over the Hotelling and monopoly rents for oil, vis-a-vis OPEC, generally focused only on reserves of oil (e.g., Pindyck, 1978).

world's oil resources although it has accelerated the depletion of other energy resources such as coal.

Finally, a basic result of the grade selection literature is that least cost reserves are exploited first. In actual fact, one sees a multiplicity of different grades simultaneously extracted. While there can be microtheoretic reasons for lower grades being produced in conjunction with higher grades (see Slade, 1988), we show that in a spatial market with costly transportation, one expects different grades of a resource to be produced at any one time.

A related issue concerns the basic definition of an economic market.<sup>2</sup> Marshall (1920), drawing on Cournot, defines a market as the area within which the price of the same good tends to uniformity, allowance being made for transportation costs. Although this basically refers to the geographic extent of the market, the definition is frequently interpreted to mean that products are in the same market when their prices move together through time (Stigler and Sherwin, 1985). We show that as long as there is some degree of substitutability between two exhaustible resources, then their rents will be related and thus are likely to be correlated over time. In a geographic context, this result can be interpreted as saying that even though two geographic markets may be distinct and non-overlapping, the sharing of a common boundary is sufficient to relate the rents throughout the two markets. This result calls into question the

---

<sup>2</sup>The concept of market or economic market as used here may not coincide with the notion of a relevant market as used in an anti-trust context (see, e.g., Scheffman and Spiller, 1987).

usefulness of price correlation for establishing an economic market for an exhaustible resource. Furthermore, the fact that rents rise over time will cause some markets to contract and some to expand their geographic scope over time.

In the next section of the paper we set up a simple linear Hotelling model of spatial competition (interpretable as product space or geographic space) with a resource deposit at each end of the line. We then explore the relationship between rents at these two deposits.

## II. THE SPATIAL MODEL

We first consider the case of a Hotelling spatial model.<sup>3</sup> As in Hotelling's (1929) original model, we consider a linear market--a straight line of unit length with consumers distributed along it. In contrast to Hotelling and much of the related literature, we are not concerned with entry of producers. We assume that there are two resource deposits, located at each end of the line segment. Production will be characterized by constant returns with constant marginal production costs. A competitive transportation industry connects producers and consumers.

---

<sup>3</sup> Although the Hotelling model was first proposed by Hotelling (1929), it has been extended by many authors, including Smithies (1941), Lerner and Singer (1937), Greenhut (1952), Eaton (1972), Hartwick and Hartwick (1971), Eaton and Lipsey (1975), and d'Aspremont et al. (1979). See Graitson (1982) or Eaton and Lipsey (1989) for recent reviews of this literature. A significant portion of this literature concerns entry which is not an issue here.



As pointed out by Hotelling (1929) and others, while this model is couched as a model in geographic space with conventional transportation, it can also be viewed as referring to product space. The goods at the two ends of the line have different levels of a characteristic. Consumers locate along the line according to the level of the characteristic which gives them maximum utility. If a consumer must take a good produced elsewhere then the utility associated with consuming that good is diminished in proportion to the distance between the producer and consumer.

For instance, consider that copper and aluminum are at the two ends of the line with the characteristic being weight/durability or some other continuously varying characteristic that has copper at one extreme and aluminum at the other. Consumers closer to the copper end obtain utility from consuming aluminum but obtain more utility from copper. Similarly, consumers at the aluminum end obtain more utility from aluminum. Consumers in the middle are indifferent between the two products. When prices are introduced, consumers may prefer to consume a distant commodity that gives less utility provided the price is right.

In the model we assume that consumers are uniformly distributed with unitary density along a line of unitary length. Consumers at each location consume at a rate according to the individual demand function  $q(p)$ . Placing the origin at the left-most end of the line, producers are assumed to be located at 0 and 1. For the time being, we assume producers are price-takers. Other notation is as follows:

- $b_i$ : average unit production cost,  $i = 0, 1$   
 $S_i$ : initial stock of resources at  $i = 0, 1$   
 $\lambda_i$ : initial resource rent at  $i = 0, 1$   
 $\tau(\cdot)$ : unit transport cost for the resource as a function of distance  
 $t$ : time  $\in [0, \infty)$   
 $M(t)$ : location of market boundary for producer 1 at time  $t$   
 $T_i$ : time of resource exhaustion for producer  $i$ ,  $i = 0, 1$   
 $T$ :  $\min(T_0, T_1)$   
 $r$ : rate of interest.

Without loss of generality, we assume that producer 0 is exhausted first (i.e.,  $T = T_0$ ). There are two basic types of regimes over the two-firm production horizon  $[0, T_0]$ . The first is that the market is always covered; i.e., all consumers are served throughout the period. The second possibility is that for some consumers delivered prices get so high that they choose not to consume. The first case is more common and is what we focus on here.

#### A. The Competitive Solution

We first consider the case that reserves at each end of the line segment are owned by many price-taking producers. At a particular point in time,  $t < T_0$ , the market boundary,  $M(t)$ , is defined by

$$b_0 + \lambda_0 e^{rt} + \tau(M) = b_1 + \lambda_1 e^{rt} + \tau(1-M) \quad (2)$$

which is a consequence of the fact that at the market boundary  $M$ , the delivered price from the two producers is equal. Implicit in this equation is that Hotelling resource rents rise at the rate of interest

(Hotelling, 1931). This also implies that two differential grades of resources ( $b_0 \neq b_1$ ) may be exploited simultaneously ( $0 < M < 1$ ). This is of course a consequence of the spatial separation of the producers. Similarly,  $T$  is the point in time where  $M$  becomes zero:

$$b_0 + \lambda_0 e^{rT} + \tau(0) = b_1 + \lambda_1 e^{rT} + \tau(1) \quad (3a)$$

$$\Rightarrow T = \frac{1}{r} \ln \left\{ \frac{b_1 + \tau(1) - [b_0 + \tau(0)]}{\lambda_0 - \lambda_1} \right\}. \quad (3b)$$

Since logarithms are only defined over non-negative rents and since we can reasonably assume that producing area 1 cannot initially undercut producing area 0 at 0 in costs, net of rent, it must be that

$$\lambda_0 > \lambda_1. \quad (4)$$

This occurs because over time, as producing area 0 moves towards exhaustion at  $T$ ,  $M$  moves towards 0. This happens because for a particular location delivered prices from 0 are rising more rapidly than from 1. Thus 0 is losing customers to 1. But this is what 0 wants: 0 wants to lose customers such that at time period  $T$  the last ton is being produced and producers at area 1 are knocking at the gates of the mine. This can also be seen by rewriting (3a) as:

$$\lambda_0 - \lambda_1 = \{b_1 + \tau(1) - [b_0 + \tau(0)]\} e^{-rT}. \quad (5)$$

This says that the rent premium enjoyed by producing area 0 is equal to the cost margin between the two producing areas, measured at producing area 0's location, discounted back to the present from the time of exhaustion of producing area 0 ( $T$ ). Thus if  $T$  is very large, rent

differences may not be great. In essence, producing area 1, by gradually eating away at producing area 0's market, is effectively reducing overall demand while simultaneously acting as a "backstop" to producing area 0's rent. Both of these effects diminish the rent of producing area 0.

This result can be summarized in the following proposition:

Proposition 1: In the competitive spatial Hotelling model with a deposit of an exhaustible resource at each end of the line segment  $[0,1]$ , with exhaustion of deposit 0 occurring first, at time  $T$ , initial rents are related as in equation (5).

The actual level of rents can only be found by integrating demand for each producer's output and setting it equal to initial stocks of resources:

$$S_0 = \int_0^T \int_0^{M(t)} q[b_0 + \lambda_0 e^{rt} + \tau(z)] dz dt \quad (6)$$

$$S_1 = \int_0^T \int_{M(t)}^1 q[b_1 + \lambda_1 e^{rt} + \tau(1-z)] dz dt \\ + \int_T^\infty \int_0^1 q[b_1 + \lambda_1 e^{rt} + \tau(1-z)] dz dt \quad (7)$$

Equations (6-7) are two equations in the two unknowns,  $\lambda_0$  and  $\lambda_1$  and thus can generally be solved for  $\lambda_0$  and  $\lambda_1$ . Unfortunately, without more structure, equations (6-7) cannot be solved analytically. Later in the paper we will simplify the problem somewhat so that equations (6-7) can be explicitly solved.

## B. The Monopoly Solution

Suppose we now consider the case where production at 0 is monopolized while production at 1 remains competitive. We will continue assuming that location 0 is exhausted first. Rent at location 0 will be assumed to include both Hotelling rent and monopoly rent and will be denoted  $\lambda_0(t)$ . Equation (2) defining the boundary between the two market areas still applies, though in a slightly modified form:

$$b_0 + \lambda_0(t) + \tau(M) = b_1 + \lambda_1 e^{rt} + \tau(1-M). \quad (2')$$

This equation implicitly defines M. Profits for the monopolist are given by

$$\pi_0 = \int_0^T \lambda(t) e^{-rt} \int_0^{M(t)} q(\lambda(t)+b_0+\tau(x)) dx dt \quad (8a)$$

which is maximized over  $\lambda(t)$  and  $R(t)$ , remaining reserves at 0, subject to the restrictions

$$\dot{R} = - \int_0^M q(\lambda(t)+b_0+\tau(x)) dx \quad (8b)$$

$$\lambda_0(T) = \lambda_1 e^{rT} + \tau(M) \quad (8c)$$

$$R(T) = 0 \quad (8d)$$

$$R(0) = S_0 \quad (8e)$$

which yields the current value Hamiltonian

$$H = [\lambda_0(t) - \mu(t)] \left[ \int_0^{M(t)} q(\lambda(t)+b_0+\tau(x)) dx \right] \quad (9)$$

for which first-order conditions with respect to  $\lambda_0$  and R are



$$\frac{\partial H}{\partial \lambda_0}: \mu(t) = \lambda_0(t) \left[ 1 + \frac{1}{\varepsilon(t)} \right] \quad (10a)$$

$$\text{where } \varepsilon(t) = \frac{d}{d \ln \lambda_0(t)} \ln \left\{ \int_0^{M(t)} q[\lambda_0(t) + b_0 + \tau(x)] dx \right\}$$

$$\frac{\partial H}{\partial R}: \frac{\dot{\mu}(t)}{\mu(t)} = r \quad (10b)$$

plus the transversality condition on  $T$ . Equation (10b) is the familiar condition that the marginal value of the resource must rise at the rate of interest. Equation (10a) relates the marginal value of the resource to the markup ( $\lambda_0(t)$ ). The  $\varepsilon(t)$  term is simply the price (rent) elasticity of aggregate demand for production from 0.

Clearly  $\varepsilon(t) < 0$ , in which case then equation (10a) implies  $\mu(t) < \lambda_0(t)$ . However, whether or not  $\dot{\lambda}_0/\lambda_0 \lesseqgtr r$  depends on  $\dot{\varepsilon}(t)$ :

$$r = \frac{\dot{\mu}(t)}{\mu(t)} = \frac{\dot{\lambda}_0(t)}{\lambda_0(t)} - \frac{\dot{\varepsilon}(t)}{\varepsilon(t)[\varepsilon(t)+1]}, \quad (11)$$

the rate at which the price elasticity changes over time. For elastic demand, the relationship between  $\dot{\lambda}_0/\lambda_0$  and  $r$  depends on the sign of  $\dot{\varepsilon}$ . If demand becomes more inelastic over time, the markup will rise faster than the rate of interest. However, if storage is costless, this will bring about intertemporal arbitrage, basically resulting in  $\dot{\varepsilon}(t) \leq 0$  or  $\dot{\lambda}/\lambda \leq r$  (see Dasgupta and Heal, 1979). This in turn implies  $\lambda_0(0)$  is at least as great for monopoly as in the competitive market and further that  $T$  is at least as great for the monopoly case as the competitive case.

### III. RECTANGULAR DEMAND

While the models discussed in the previous section have the obvious advantage of being general, it is for that reason that it is difficult to obtain definitive comparative statics results. Other authors have adopted restrictive assumptions on demand. Hotelling's original spatial model assumed totally inelastic demand. This of course will not work with an exhaustible resource since rents will be bid infinitely high. Eaton (1972) and Smithies (1941) assume linear demand while Lerner and Singer (1937) assume rectangular demand; i.e., inelastic demand up to a "backstop" or "choke" price at which point demand drops to zero. This later assumption is particularly common in exhaustible resource markets where there is some backstop price at which a non-exhaustible substitute enters.

We thus adopt the assumption of rectangular demand in this section of the paper. Demand is uniformly distributed over the  $[0,1]$  line segment with each consumer's demand being for one unit of the resource per unit time at prices below the backstop,  $\bar{p}$ .

We also adopt the common convention (e.g., Eaton and Lipsey, 1975; Novshek, 1980; Graitson, 1982) that transportation cost is a linear function of distance  $d$ :  $\tau(d) = \alpha d$ .

We assume reserves and costs are such that the market is always covered as long as both producers are in the market; i.e., we do not have the situation where both areas are producing with a gap in the middle of the line segment where neither area is able to supply at a price less than the backstop. Furthermore, producing area 0 has

insufficient reserves and/or high enough costs that it never serves the entire market. The result of this assumption is that there are three natural time segments. In segment  $[0, T_0]$ , both areas are producing and the entire market is served. At time  $T_0$ , area 0 is exhausted although location 0's price at  $T_0$  need not be the backstop price. In fact  $T_0$  is the point in time at which area 1 can deliver to 0 at the price area 0 is offering FOB (i.e., without transportation). The second time slice is  $[T_0, T_{01}]$  and corresponds to the time where area 1 is supplying the entire market. Finally the period  $[T_{01}, T_1]$  corresponds to time period where area 1 serves only part of the market and in fact just exhausts his resources at  $T_1$ .

#### A. A Competitive Market

The first market we consider involves multiple producers at each end of the line segment  $[0, 1]$ , with each producer a price-taker. The first question is to define the market boundary for producing region 1,  $M(t)$ . This can be obtained from equation (2):

$$M(t) = \begin{cases} \frac{b_1 - b_0 + (\lambda_1 - \lambda_0)e^{rt} + \alpha}{2\alpha} & 0 \leq t \leq T_0 \\ 0 & T_0 \leq t \leq T_{01} \\ \frac{b_1 + \lambda_1 e^{rt} - \bar{p} + \alpha}{\alpha} & T_{01} \leq t \leq T_1 \\ 1 & t > T_1 \end{cases} \quad (12)$$

$T_0$ ,  $T_{01}$ , and  $T_1$  can similarly be obtained from equation (2) or, for the case of  $T_{01}$ , when the right-hand side of (2) equals the choke price  $\bar{p}$ :

$$T_0 = \frac{1}{r} \ln \frac{b_1 + \alpha - b_0}{\lambda_0 - \lambda_1} \quad (13a)$$

$$T_{01} = \frac{1}{r} \ln \frac{\bar{p} - b_1 - \alpha}{\lambda_1} \quad (13b)$$

$$T_1 = \frac{1}{r} \ln \frac{\bar{p} - b_1}{\lambda_1} \quad (13c)$$

Equations (6-7) can now be written for this case:

$$S_0 = \int_0^{T_0} M(t) dt = \frac{1}{2\alpha r} \{ (b_1 - b_0 + \alpha) \ln \left[ \frac{b_1 - b_0 + \alpha}{\lambda_0 - \lambda_1} \right] - \alpha + b_0 + \lambda_0 - b_1 - \lambda_1 \} \quad (14a)$$

$$\begin{aligned} S_1 &= \int_0^{T_0} [1 - M(t)] dt + \int_{T_0}^{T_{01}} dt + \int_{T_{01}}^{T_1} [1 - M(t)] dt \\ &= \int_0^{T_1} dt - \int_0^{T_0} M(t) dt - \int_{T_{01}}^{T_1} M(t) dt \end{aligned} \quad (14b)$$

Equation (14a) may now be substituted into equation (14b) to obtain:

$$\begin{aligned} S_1 &= \int_0^{T_1} dt - S_0 - \int_{T_{01}}^{T_1} M(t) dt \\ &= \frac{1}{r} \ln \frac{\bar{p} - b_1}{\lambda_1} - S_0 - \frac{1}{\alpha r} \{ (b_1 - \bar{p} + \alpha) \ln \frac{(\bar{p} - b_1)}{(\bar{p} - b_1 - \alpha)} + \alpha \} \end{aligned} \quad (14c)$$

$$\Rightarrow \ln \lambda_1 = -(S_0 + S_1)r - 1 + \frac{1}{\alpha} \{ (\bar{p} - b_1) \ln (\bar{p} - b_1) - (\bar{p} - b_1 - \alpha) \ln (\bar{p} - b_1 - \alpha) \} \quad (15)$$

A comparative statics question naturally arises as to the effect of transportation costs on rents. As we increase  $\alpha$ , moving from a less spatial market to a more highly spatial one, markets shrink (the

backstop becomes "closer") which drives down rents while competition between producing areas is reduced. Totally differentiating (14a) and (15) with respect to  $\alpha$ ,  $\lambda_0$ , and  $\lambda_1$  and using (15) directly, yields respectively

$$\left(\frac{d\lambda_0}{d\alpha} - \frac{d\lambda_1}{d\alpha}\right) = \frac{\{2rS_0 - \ln[\frac{b_1 + \alpha - b_0}{\lambda_0 - \lambda_1}]\}(\lambda_0 - \lambda_1)}{(\lambda_0 + b_0 - b_1 - \lambda_1 - \alpha)} \quad (16a)$$

$$\frac{d\lambda_1}{d\alpha} = \frac{\lambda_1}{\alpha} \left\{ 1 + \frac{(\bar{p} - b_1)}{\alpha} \ln\left[\frac{\bar{p} - b_1 - \alpha}{\bar{p} - b_1}\right] \right\}. \quad (16b)$$

In our case it must be that  $\bar{p} - b_1 > \alpha$ , otherwise producers at 1 could never deliver to producers at 0 below the choke price  $\bar{p}$ . Thus<sup>4</sup> in (16b),  $d\lambda_1/d\alpha < 0$ . As transportation costs increase, the rent for producers at 1 decreases. Maximum rent occurs when  $\alpha = 0$ . Unfortunately, equation (16a) does not allow us to make a definitive conclusion about the sign of  $d\lambda_0/d\alpha$  although we suspect it to be negative. We can summarize this result in the following proposition:

Proposition 2: Assume a spatial Hotelling model with deposits of an exhaustible resource at each end of the unit interval, with rectangular demand and costs and resource stocks such that the entire market is covered while both producers are in the market. Without loss of generality, assume producers at 0 are exhausted first. Then for producers at 1,

---

<sup>4</sup>The expression in braces in (16b) is  $1 + [\ln(1-x)]/x$  which is always negative for  $0 < x < 1$ .



- (a) Hotelling rents are less than they would be if space were ignored (costless transport), with the two resource deposits treated as one; and
- (b) increases in transportation costs lead to reductions in Hotelling rent.

The implications of this proposition are quite fundamental. In a spatial market, rents are not just determined by demand and supply within the area served by a producing region. Rather, ignoring space and looking at rents that would be realized from aggregating all demand and supply places an upper bound on rent. In essence, a producing area with a large resource endowment depresses rents far beyond its current market area.

#### B. One Producing Region Monopolized

We now consider the case where producing area 0 is monopolized. All other assumptions remain as above. Thus there are multiple price-taking producers at 1 and a single price-making producer at 0. Let the price net of costs for producer 0 at the mine be  $\lambda(t)$ . Thus  $\lambda(t)$  includes monopoly rents and scarcity rents. This is the "markup" that the monopolist applies to his resource. Let  $\lambda_0 = \lambda(0)$ . The market boundary is defined as:

$$M(t) = \begin{cases} \frac{b_1 - b_0 + \lambda_1 e^{rt} - \lambda(t) + \alpha}{2\alpha} & 0 \leq t \leq T_0 \\ 0 & T_0 \leq t \leq T_{01} \\ \frac{b_1 + \lambda_1 e^{rt} - \bar{p} + \alpha}{\alpha} & T_{01} \leq t \leq T_1 \\ 1 & t > T_1 \end{cases} \quad (17)$$

where  $T_0$  is the point where producer 0 is exhausted,  $T_{01}$  is the point where producers at 1 are no longer able to supply 0 below the choke price  $\bar{p}$  and  $T_1$  is the point where producers at 1 are exhausted (by assumption  $T_0 < T_{01} < T_1$ ). It is more difficult to solve equation (2) for  $T_0$  in this case because we do not know how  $\lambda(t)$  evolves over time. We defer defining  $T_0$ ,  $T_{01}$ , and  $T_1$ .

Producer 0, as a monopolist, seeks to maximize profits by choosing a time path of markups,  $\lambda(t)$ . The current-value Hamiltonian, from (9), is

$$H = [\lambda(t) - \mu(t)] \left\{ \frac{b_1 - b_0 + \lambda_1 e^{rt} - \lambda(t) + \alpha}{2\alpha} \right\} \quad (18)$$

for which first-order conditions are

$$\frac{\partial H}{\partial \lambda}: [\lambda(t) - \mu(t)] \left[ -\frac{1}{2\alpha} \right] + \left[ \frac{b_1 - b_0 + \lambda_1 e^{rt} - \lambda(t) + \alpha}{2\alpha} \right] = 0 \quad (19a)$$

$$\frac{\partial H}{\partial R}: \dot{\mu}(t) - r\mu(t) = 0. \quad (19b)$$

Equation (19a) may be solved for  $\mu(t)$ :

$$\mu(t) = 2\lambda(t) + b_0 - (b_1 + \alpha + \lambda_1 e^{rt}). \quad (20)$$

This equation defines the marginal value to the monopolist of the resource ( $\mu(t)$ ) which, according to equation (19b) must rise at the rate of interest. Combining equation (19b) and (20) we obtain

$$\frac{d \ln \left[ \lambda(t) - \frac{b_1 + \alpha - b_0}{2} \right]}{dt} = r \quad (21)$$

which can be solved for  $\lambda(t)$ :

$$\lambda(t) = \left\{ \lambda_0 - \frac{b_1 + \alpha - b_0}{2} \right\} e^{rt} + \frac{b_1 + \alpha - b_0}{2} \quad (22)$$

Since  $b_1 + \alpha > b_0$ , equation (22) tells us that  $\lambda(t)$  rises at a rate less than the rate of interest,  $r$ .

We are now in a position to use equation (2') to define  $T_0$ ,  $T_{01}$ , and  $T_1$ :

$$T_0 = \frac{1}{r} \ln \left\{ \frac{b_1 + \alpha - b_0}{2(\lambda_0 - \lambda_1) + b_0 - b_1 - \alpha} \right\} \quad (23a)$$

$$T_{01} = \frac{1}{r} \ln \frac{\bar{p} - b_1 - \alpha}{\lambda_1} \quad (23b)$$

$$T_1 = \frac{1}{r} \ln \frac{\bar{p} - b_1}{\lambda_1} . \quad (23c)$$

Note that the expressions for  $T_{01}$  and  $T_1$  are identical in form to the competitive case (equation (13)) although the times may be different since  $\lambda_1$  may be different.

Conditions that stocks balance flows can now be specified

$$\int_0^{T_0} M(t) dt = S, \quad (24a)$$

$$\begin{aligned} \int_0^{T_0} (1-M(t)) dt + \int_{T_0}^{T_{01}} 1 dt + \int_{T_{01}}^{T_1} (1-M(t)) dt \\ = \int_0^{T_1} 1 dt - \int_0^{T_0} M(t) dt - \int_{T_{01}}^{T_1} M(t) dt = S_1 \end{aligned} \quad (24b)$$

Equation (24a) may be substituted into equation (24b) to obtain

$$S_1 = \int_0^{T_1} dt - S_0 - \int_{T_{01}}^{T_1} M(t)dt. \quad (24c)$$

Note however that the expressions for  $T_{01}$  and  $T_1$  are identical in the monopoly case to the competitive case (compare equations (24b-c) to equations (13b-c)). Furthermore,  $M(t)$ , for  $T_{01} \leq t \leq T_1$ , is identical in the two cases (compare equation (17) with equation (12)). Thus, equation (24c) is identical to equation (14c) which implies  $\lambda_1$  is identical and thus is as given by equation (15). Thus, whether or not producing area 0 is a monopolist has no effect on the scarcity rent at 1.

We are now in a position to examine  $\lambda_0$ . We noted from equation (22) that  $\lambda(t)$  rises more slowly than the rate of interest. Since prices for producers at 1 are the same for the monopoly case as the competitive case, it must be that  $\lambda_0$  is elevated in the monopoly case over what would prevail in the competitive case. Otherwise producer 0 would produce more than  $S_0$ . We summarize these results in the following proposition:

Proposition 3: Let the assumptions of Proposition 2 apply except that producing area 0 is monopolized. Then

- (a) The fact that producer 0 is a monopolist does not effect the Hotelling rents for (price-taking) producers at 1.
- (b) The initial price for producer 0 (net of transport) is elevated relative to the competitive case. Furthermore, the price net of costs rises at a rate less than the rate of interest.

- (c) Monopolization of producing area 0 results in accelerated depletion of producing area 1 initially followed by some conservation of the resources of producing area 1 (compared to the competitive case).
- (d) The time to exhaustion of both deposits is unaffected by the fact that producer 0 is a monopolist.

This proposition has a number of rather startling implications. Monopolization of one producing area does not eliminate rents in separate areas. We thus have the result from Proposition 1 that scarcity rents at the two deposits are related yet extraction of monopoly rent at one deposit will not affect scarcity rents at other locations. In effect, monopoly at one location results in an initially smaller market area for the monopolist (relative to competition), with increased inroads by other producing areas. But because the monopolist's market area shrinks more slowly than in the competitive case, during some later period of time the monopolist's market area is actually larger than in the competitive case with exhaustion of the monopolist's deposit delayed. Relative to the competitive case, the other producing area initially produces more and later produces less with the overall time of depletion of all deposits remaining the same in the two cases. Thus the maxim that "monopoly is the conservationist's best friend" must be modified. Monopoly does depress initial production by the monopolist but it accelerates initial production from others. The overall time to complete exhaustion is the same with monopoly and pure competition. Of course it goes without saying that these results do depend on our assumptions.



### C. Extensions to Product Space

The discussion in the previous two sections was confined to geographic space with  $\alpha$  representing true unit transportation costs. However, as Hotelling recognized in his 1929 paper, and as is discussed earlier in this paper, space may also be interpreted as product space. Transport costs are analogous to the monetary value of disutility where goods transported further generate less utility.

Consider two exhaustible resources which are substitutes but not perfect substitutes. They are on opposite ends of a unit interval in product space (e.g., coal and oil or copper and aluminum). Proposition 1 states that rents associated with one exhaustible resource depend on the scarcity of the other, even if the two appear currently to be in distinct markets. Thus the scarcity value of oil depends not only on how much oil is available but on how much coal. Even if the resources are in separate uses now, the net price of the scarcer resource will rise faster; thus as time proceeds the two resources move closer and closer together in the minds of consumers. If this price of oil were far higher than it is today, coal would be used in many applications that are currently the sole province of oil. That is why the rents are coupled. Attempts that have been made to estimate the scarcity value of oil have erred to the extent that they ignored other exhaustible resources.

Proposition 2 states that the fact that people discriminate between the two resources depresses the scarcity rent associated with the less scarce resource. Thus the fact that some people prefer oil to coal (all other things being equal) rather than just desiring

energy in general (regardless of its form) results in a lower scarcity rent for coal.

Proposition 3 states that monopoly of one resource will not effect scarcity rents for other substitute resources despite the fact that monopoly will accelerate near-term production of substitutes. This will be offset later by higher production of the monopolized resource. Thus a monopoly in the oil market should not affect scarcity rents for coal even though demand for coal may be increased in the near-term.

Of course all of these results are predicated on the underlying assumptions of the analysis. Propositions 2 and 3 in particular depend on the assumption of rectangular demand which is probably one of the simplest representations of demand--valid in some cases and off-the-mark in others.

#### IV. CONCLUSIONS

This paper has examined the interrelationship between resource rents in related exhaustible resource markets. Virtually all work to date on Hotelling rents has examined an exhaustible resource in isolation, taking into account substitutes only via the demand curve for the resource. However if substitutes are also exhaustible then demand is not exogenous but contains terms (rents of the substitutes) that are determined simultaneously with supply/rent of the exhaustible resource.

In a product space context, we have shown that even if two exhaustible resources are not currently in the same market (e.g., coal and oil), different levels of scarcity will cause them to be in the same

market at some point in the future which in turns links their current scarcity rents. Thus ignoring substitutes for an exhaustible resource overstates rents. A natural extension of this is that current estimates of scarcity rents may be too high. Another result is that while monopoly for one exhaustible resource may initially conserve that resource, the monopoly accelerates depletion of substitutes. Furthermore, under plausible conditions, monopoly does not conserve the resource over the long run. Ultimate depletion times are unaffected by monopolization.

The markets for many exhaustible resources are highly spatial, due to the bulky nature of the commodities and relatively high transport cost-value ratios (e.g., coal). Such resources are typically viewed as being supplied in separated geographic markets. For instance, a western coal market might be considered to be separate from an eastern coal market. Our results indicate that despite current separation of markets, differential endowments of resources mean that market boundaries will shift over time. Thus rents in separate markets are related. For a particular deposit, the presence of large deposits in distant locations can and will depress local Hotelling rents. Even if two deposits appear to be in separate markets today, if at some future point in time the two deposits are in the same market, then today's Hotelling rents are related. If one of the two deposits is monopolized, the effect will be a higher initial price for the monopolized deposit and thus a smaller initial geographic market. However, given our assumption on the market, overall time to exhaustion is not

affected since the monopolist's market shrinks more slowly than in the competitive case.

Clearly the interaction among exhaustible resource markets requires a modification in our views of exhaustible resource markets. This paper is only the start of examining this question.

V. REFERENCES

- Dasgupta, P. S. and G. M. Heal, Economic Theory and Exhaustible Resources (Cambridge University Press, Cambridge, 1979).
- D'Aspremont, C., J. Jaskold Gabszewicz and J.-F. Thisse, "On Hotelling's 'Stability in Competition'," Econometrica, 47: 1145-50 (1979).
- Eaton, B. Curtis, "Spatial Competition Revisited," Can. J. Econ., 5: 268-77 (1972).
- Eaton, B. Curtis and Richard G. Lipsey, "The Principle of Minimum Differentiation Reconsidered: Some New Developments in the Theory of Spatial Competition," Rev. Econ. Stud., 42: 27-49 (1975).
- Eaton, B. Curtis and Richard G. Lipsey, "Product Differentiation," in R. Schmalensee and R. D. Willig (eds.), Handbook of Industrial Organization (North-Holland, Amsterdam, 1989).
- Graitson, Dominique, "Spatial Competition a la Hotelling: A Selective Survey," J. Industrial Econ., 31: 13-25 (1982).
- Hartwick, John M. and Philip G. Hartwick, "Duopoly in Space," Can. J. Econ., 4: 485-505 (1971).
- Hotelling, Harold, "Stability in Competition," Economic Journal, 39: 41-57 (1929).
- Hotelling, Harold, "The Economics of Exhaustible Resources," J. Pol. Econ., 39: 137-75 (1931).
- Lerner, A. B. and H. W. Singer, "Some Notes on Duopoly and Spatial Competition," J. Pol. Econ., 45: 145-86 (1937).
- Marshall, Alfred, Principles of Economics, Eighth Ed. (1920).
- Novshek, William, "Equilibrium in Simple Spatial (or Differentiated Product) Models," J. Econ. Theory, 22: 313-26 (1980).
- Pindyck, Robert S., "Gains to Producers from the Cartelization of Exhaustible Resources," Rev. Econ. and Statistics, 60: 238-51 (1978).
- Scheffman, David T. and Pablo T. Spiller, "Geographic Market Definition Under the U.S. Department of Justice Merger Guidelines," J. Law and Econ., 30: 123-47 (1987).
- Slade, Margaret, "Grade Selection under Uncertainty: Least Cost Last and Other Anomalies," J. Env. Econ. and Mgmt., 15: 189-205 (1988).

- Smithies, Arthur, "Optimum Location in Spatial Competition," J. Pol. Econ., 49: 423-39 (1941).
- Stigler, George J. and Robert A. Sherwin, "The Extent of the Market," J. of Law and Econ., 28: 555-85 (1985).



- No. 1 Susan I. Cohen. "Pareto Optimality and Bidding for Contracts" Working Paper # 1411
- No. 2 Jan K. Brueckner and Kangoh Lee. "Spatially-Limited Altruism, Mixed Clubs, and Local Income Redistribution" Working Paper #1406
- No. 3 George E. Monahan and Vijay K. Vemuri. "Monotonicity of Second-Best Optimal Contracts" Working Paper #1417
- No. 4 Charles D. Kolstad, Gary V. Johnson, and Thomas S. Ulen. "Ex Post Liability for Harm vs. Ex Ante Safety Regulation: Substitutes or Complements?" Working Paper #1419
- No. 5 Lanny Arvan and Hadi S. Esfaheni. "A Model of Efficiency Wages as a Signal of Firm Value" Working Paper #1424
- No. 6 Kalyan Chatterjee and Larry Samuelson. "Perfect Equilibria in Simultaneous-Offers Bargaining" Working Paper #1425
- No. 7 Jan K. Brueckner and Kangoh Lee. "Economies of Scope and Multiproduct Clubs" Working Paper #1428
- No. 8 Pablo T. Spiller. "Politicians, Interest Groups, and Regulators: A Multiple-Principals Agency Theory of Regulation (or "Let Them Be Bribed" Working Paper #1436
- No. 9 Bhaskar Chakravorti. "Asymmetric Information, 'Interim' Equilibrium and Mechanism Design" Working Paper #1437
- No. 10 Bhaskar Chakravorti. "Mechanisms with No Regret: Welfare Economics and Information Reconsidered" Working Paper #1438
- No. 11 Bhaskar Chakravorti. "Communication Requirements and Strategic Mechanisms for Market Organization" Working Paper #1439
- No. 12 Susan I. Cohen and Martin Loeb. "On the Optimality of Incentive Contracts in the Presence of Joint Costs" Working Paper #1445
- No. 13 Susan I. Cohen and Martin Loeb. "The Demand for Cost Allocations: The Case of Incentive Contracts Versus Fixed-Price Contracts" Working Paper #1455
- No. 14 Jan K. Brueckner and Kevin M. O'Brien. "Modeling Government Behavior in Collective Bargaining: A Test for Self-Interested Bureaucrats" Working Paper #1481
- No. 15 Jan K. Brueckner. "Estimating a Bargaining Contract Curve: Prior Restrictions and Methodology" Working Paper #1490
- No. 16 Peter C. Reiss and Pablo T. Spiller. "Competiton and Entry in Small Airline Markets" Working Paper #1497
- No. 17 Pablo T. Spiller. "A Note on Pricing of Hub-and-Spoke Networks" Working Paper #1498
- No. 18 Larry DeBrock. "Joint Marketing Efforts and Pricing Behavior" Working Paper #1500
- No. 19 Frank A. Wolak and Charles D. Kolstad. "A Model of Homogenous Input Demand Under Price Uncertainty" Working Paper #1502
- No. 20 Susan I. Cohen. "Reputation, Intertemporal Incentives and Contracting" Working Paper #1511
- No. 21 Lenny Arvan and Antonio Leite. "A Sequential Equilibrium Model of Cost Overruns in Long Term Projects" Working paper #1514
- No. 22 Jan K. Brueckner and Pablo T. Spiller. "Competiton and Mergers in Airline Networks" Working Paper #1523
- No. 23 Hadi S. Esfahani. "Reputation, Product Quality, and Production Technology in LDC Markets" Working Paper #89-1525
- No. 24 Hadi S. Esfahani. "Moral Hazard, Limited Entry Costs, and 'Introductory Offers'" Working Paper #89-1526
- No. 25 Bhaskar Chakravorti. "Mechanisms with No Regret: Welfare Economics and Information Reconsidered" Working Paper #89-1527
- No. 26 Susan I. Cohen. "Implicit Cost Allocation and Bidding for Contracts" Working Paper #89-1558
- No. 27 Rafael Gely and Pablo T. Spiller. "A Rational Choice Theory of the Supreme Court" Working Paper #89-1559
- No. 28 Rafael Gely and Pablo T. Spiller. "An Economic Theory of Supreme Court Statutory Decisions The State Farm and Grove City Cases" Working Paper #89-1560
- No. 29 Rafael Gely and Pablo T. Spiller. "The Political Economy of Supreme Court Constitutional Decisions: The Case of Roosevelt's Court Packing Plan" Working Paper #89-1561
- No. 30 Hadi S. Esfahani. "Reputation and Product Quelity Revisited." Working Paper #89-1584
- No. 31 Jan K. Brueckner. "Growth Controls and Land Values in an Open City." Working Paper #89-1594
- No. 32 Jan K. Brueckner. "Tastes, Skills, and Local Public Goods." Working Paper #89-1610
- No. 33 Luis Cabral and Shane Greenstein. "Switching Costs and Bidding Parity in Government Procurement of Computer Systems." Working Paper #90-1628
- No. 34 Charles D. Kolstad. "Hotelling Rents in Hotelling Space: Exhaustible Resource Rents with Product Differentiation." Working Paper #90-1629







ECKMAN  
NDERY INC.



JUN 95

To-Place® N. MANCHESTER,  
INDIANA 46962



UNIVERSITY OF ILLINOIS-URBANA



3 0112 042686953